N-Body Simulations

Sverre Aarseth

Institute of Astronomy, Cambridge

The Basic Problem

Model Ingredients

Hardware & Software

Code Tests

Performance

Newton's Equation

Force $\quad \mathbf{F}_i = -G \sum\limits_{j=1; \, j \neq i}^{N} m_j \dfrac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}$

Explicit differentiation

$$\dot{\mathbf{F}}_i = -G \sum\limits_{j=1; \, j \neq i}^{N} m_j \dfrac{\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

$$- 3m_j \dfrac{(\mathbf{r}_i - \mathbf{r}_j) \cdot (\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_j)}{|\mathbf{r}_i - \mathbf{r}_j|^2} \dfrac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

New solution at $t = \Delta t$

$$\Delta \dot{\mathbf{r}}_i = (\tfrac{1}{2} \dot{\mathbf{F}}_i \Delta t + \mathbf{F}_i) \Delta t$$

$$\Delta \mathbf{r}_i = ((\tfrac{1}{6} \dot{\mathbf{F}}_i \Delta t + \tfrac{1}{2} \mathbf{F}_i) \Delta t + \dot{\mathbf{r}}_i) \Delta t$$

Repeat cycle for $i = 1, N$; $\quad N^2$ interactions

Model Ingredients

System type	Planets or star clusters
Initial masses	$n(m) \propto m^{-\alpha}$
Primordial binaries	Periods & mass ratios
Stellar evolution	HR diagram & mass loss
External effects	Galactic tidal field
Post-Newtonian	Black holes or neutron stars
Cosmology	$E_{\text{tot}} > 0, \quad \mathbf{V}_i \propto H\mathbf{R}_i$

Hardware

IBM 7090	1962, London IBM Centre
IBM 360-44	1967, Institute Theoretical Astronomy
MIPS	1989, grant GBP 24,000
micro-VAX	1989, Institute of Astronomy
HARP-2	1994, 8 pipes, special-purpose
HARP-3	1995, grant, GBP 48,000
GRAPE-6	2001, 88 pipes, grant GBP 75,000
micro-GRAPE	2004, grant GBP 5,000
GPU	2008, grant GBP 1500

Software

Challenges	Δt, F & long time-scale
Force polynomial	$\mathbf{F}(t)$, divided differences
Individual time-steps	Taylor series convergence
Neighbour scheme	$\mathbf{F}_i = \mathbf{F}_{\mathrm{irr}} + \mathbf{F}_{\mathrm{reg}}$
Hermite method	Self-starting 4th order
Block time-steps	Hierarchical levels
Close encounters	Two-body regularization
Compact subsystems	Chain regularization

Programming

NBODY6	330 routines and 56,000 lines
Languages	FORTRAN, C++ & CUDA
Basic variables	70 size N arrays & size N lists
Parameters	$NMAX$ before compilation
Common blocks	common6.h, arrays & scalars
Subroutines	FORTRAN, C++ & GPU
Coding style	6 letter variables, verbal comments
Options	More than 40, multiple values
Downloads	www.ast.cam.ac.uk/research/nbody

```
*         common6.
*         -------
*
      INCLUDE 'params.h'
      IMPLICIT REAL*8  (A-H,O-Z)
      REAL*8  MP,MP0,MPDOT
*
      COMMON/NAMES/   NTOT,NPAIRS,NTTOT,N,NNBMAX,NCRIT,NFIX,NMERGE,NSUB,
     &                IPHASE,IFIRST,ICOMP,JCOMP,ICLOSE,JCLOSE,JCMAX,
     &                KSPAIR,NRUN,MODEL,NC,NZERO,NBZERO,NBIN0,NHI0,
     &                NAME0,NCH,NCHAOS,IDUM1,KZ(50),NNBOPT,NEW2(8)

      COMMON/PARAMS/  ETAI,ETAR,DTADJ,DELTAT,TCRIT,QE,RBAR,ZMBAR,
     &                DTMIN,RMIN,ETAU,CMSEP2,ECLOSE,GMIN,GMAX,ETA0,
     &                TWOPI,ONE3,ONE6,ONE9,ONE12,TCR0,TRH,BODYM,BODY1,
     &                SMIN,RMIN2,RMIN22,STEPJ,ALPHA,ZNBMIN,ZNBMAX,EBH,
     &                TIME,TADJ,TNEXT,CPU,CPU0,CPUTOT,ZMASS,RSCALE,TCR,
     &                TRC,BE(3),CMR(4),CMRDOT(4),ZKIN,POT,EBIN,EBIN0,
     &                ESUB,EMERGE,ECOLL,EDISS,ESYNC,E(12),ERROR,ERRTOT,
     &                DETOT,ETCORR,AZ,PCRIT,EBCH0,RTIDE,TSCALE,TIDAL(4),
     &                HT,ETIDE,EGRAV,RSFAC,RSPH2,RC,RC2,RC2IN,VC,ZMC,
     &                RDENS(3),RHOD,RHOM,RSMIN,RMAX,DMIN1,DMIN2,DMIN3,
     &                DMIN4,DMINC,SBCOLL,BBCOLL,CHCOLL,DELTAS,ORBITS(9),
     &                GPRINT(9),TLASTT,TLASTS,TLASTB(9),TDUMP,
     &                SCOEFF(12),TOFF,TTOT,EBESC,EMESC,ESESC,CLIGHT,
     &                RZ,TINY,SMAX,WTOT,WTOT0,VRMS,DUMMY(97)

      COMMON/COUNTS/  NSTEPI,NSTEPR,NSTEPU,NNPRED,NBCORR,NBFULL,NBVOID,
     &                NNTB,NBSMIN,NLSMIN,NBDIS,NBDIS2,NCMDER,NBDER,
     &                NFAST,NBFAST,NBLOCK,NBPRED,NICONV,NCHAIN,NSTEPC,
     &                NKSTRY,NKSREG,NKSHYP,NKSPER,NPRECT,NEWKS,NKSMOD,
     &                NTTRY,NTRIP,NQUAD,NMERG,NSTEPT,NSTEPQ,NDISS,NTIDE,
     &                NCOLL,NSYNC,NSESC,NBESC,NMESC,NTIMER,NSTEPS,NPRINT,
     &                NDUMP,NBPREV,NEWHI,NSTEPB,NBFLUX,NMTRY,NWARN,
     &                NIRECT,NURECT,NBRECT,NRRECT,KSMAG,NOFL(2),NPOP(10),
     &                NBLCKR,NDUMMY(99)

      COMMON/PLPOT/   MP,AP2,VIR,MP0,MPDOT,TDELAY,RTIDE0,QVIR,PLDUM(4)

      COMMON/BLOCKS/  TPREV,TBLOCK,DTK(40),KVEC(2*KMAX)

      COMMON/STARS/   EPOCH0,ZMRG,ZMHE,ZMRS,ZMWD,ZMSN,ZMNH,ZMBH,ZMDOT,
     &                AU,PC,GM,DAYS,YRS,SU,SMU,RAU,TSTAR,VSTAR,STEPX,
     &                TMDOT,TPHYS,TURN,EMDOT,ECDOT,EKICK,TPLOT,DTPLOT,
     &                XHYD,YHEL,ZMET,ZPARS(20),SPNFAC,IQCOLL,NAS,NBH,
     &                NBKICK,NBR,NBRK,NBS,NCHA,NCIRC,NCOAL,NCONT,NDD,
     &                NEMOD,NGB,NGLOB,NGLOB0,NHE,NHG,NHI,NHYP,NKICK,
     &                NMDOT,NMS,NNH,NRG,NRO,NROCHE,NRS,NRSAVE,NSHOCK,
     &                NSLP,NSN,NSP,NSPIR,INSTAB,NTZ,NWD,NCE,NHYPC,NBH0,
     &                ITYPE(5),KSAVE(4),KTYPE(0:14,0:14),NEINT,IBLUE,
     &                ITAIL0,NTAIL,NTDUM,NSTAIL,N1,NGDUM(7),
     &                LISTR(MLR),LISTD(MLD),LISTV(MLV)
```

```
      COMMON/NBODY/   X(3,NMAX),X0(3,NMAX),X0DOT(3,NMAX),F(3,NMAX),
     &                FDOT(3,NMAX),BODY(NMAX),RS(NMAX),XDOT(3,NMAX),
     &                FI(3,NMAX),D1(3,NMAX),D2(3,NMAX),D3(3,NMAX),
     &                FR(3,NMAX),D1R(3,NMAX),D2R(3,NMAX),D3R(3,NMAX),
     &                STEP(NMAX),T0(NMAX),STEPR(NMAX),T0R(NMAX),
     &                TNEW(NMAX),RADIUS(NMAX),TEV(NMAX),TEV0(NMAX),
     &                BODY0(NMAX),EPOCH(NMAX),SPIN(NMAX),XSTAR(NMAX),
     &                ZLMSTY(NMAX),FIDOT(3,NMAX),D0(3,NMAX),
     &                FRDOT(3,NMAX),D0R(3,NMAX),KSTAR(NMAX)

*
      COMMON/PAIRS/   U(4,KMAX),U0(4,KMAX),UDOT(4,KMAX),FU(4,KMAX),
     &                FUDOT(4,KMAX),FUDOT2(4,KMAX),FUDOT3(4,KMAX),
     &                H(KMAX),HDOT(KMAX),HDOT2(KMAX),HDOT3(KMAX),
     &                HDOT4(KMAX),DTAU(KMAX),TDOT2(KMAX),TDOT3(KMAX),
     &                R(KMAX),R0(KMAX),GAMMA(KMAX),SF(7,KMAX),H0(KMAX),
     &                FP0(4,KMAX),FD0(4,KMAX),TBLIST,DTB,KBLIST(KMAX),
     &                KSLOW(KMAX),NAME(NMAX),LIST(LMAX,NMAX)
*
      COMMON/LISTS/   ILIST(NMAX),JLIST(NMAX),JPERT(5*LMAX)
```

```
SOURCE = \
nbody6.f adjust.f assess.f bindat.f binev.f binout.f binpop.f \
block.f bodies.f brake.f brake2.f brake3.f bsetid.f chaos0.f chaos.f \
check.f checkl.f chrect.f clint.f cloud.f cloud0.f cmbody.f \
cmcorr.f cmfirr.f cmfreg.f coal.f comenv.f core.f corerd.f \
cputim.f data.f decide.f define.f deform.f degen.f delay.f \
dgcore.f dtchck.f eccmod.f ecirc.f edot.f efac2.f efac3.f \
expel.f energy.f escape.f events.f evolve.f expand.f fclose.f \
fcloud.f fcorr.f fdisk.f fhalo.f ficorr.f findj.f findm.f \
flyby.f fnuc.f fpcorr.f fpert.f fpoly1.f fpoly2.f freeze.f \
gcinit.f gcint.f giant.f giant3.f gntage.f grrad.f hcorr.f \
hiarch.f hicirc.f hidat.f higrow.f himax.f himax2.f himod.f \
hipop.f hirect.f histab.f hivel.f hmdot.f hmdot2.f hotsys.f \
hrdiag.f hrplot.f hut.f hut2.f iblock.f imf.f imfbd.f imf2.f \
impact.f induce.f inext.f input.f insert.f instar.f intgrt.f \
intide.f jacobi.f kepler.f kick.f kick2.f ksapo.f kscorr.f \
ksin2.f ksinit.f ksint.f kslist.f ksmod.f ksperi.f kspert.f \
kspoly.f kspred.f ksrect.f ksreg.f ksres.f ksres2.f ksterm.f \
kstide.f lagr.f lagr2.f levels.f magbrk.f matrix.f mdot.f merge.f \
merge2.f mix.f mloss.f mlwind.f modify.f mrenv.f mtrace.f mydump.f \
nbhist.f nbint.f nblist.f nbpot.f nbrem.f nbrest.f nbsort.f nbtide.f \
newtev.f nstab.f ntint.f offset.f orbit.f output.f peri.f permit.f \
pfac.f poti.f proto_star.f qtides.f ran2.f reflct.f regint.f \
remove.f rename.f reset.f reset2.f resolv.f rkint.f rl.f roche.f \
rpmax.f rpmax2.f rpmin.f scale.f search.f setup.f setup2.f shrink.f \
sort1.f spiral.f stability.f star.f start.f stepk.f steps.f stumpf.f \
subint.f swap.f sweep.f synch.f tail0.f tcirc.f tides.f tides2.f \
tides3.f touch.f tpert.f trdot.f trdot2.f trflow.f tstab.f tstep.f \
units.f unpert.f update.f verify.f xtrnl0.f xtrnld.f xtrnlf.f xtrnlp.f \
xtrnlt.f xtrnlv.f xvpred.f zare.f zcnsts.f zero.f zfuncs.f \
triple.f derqp3.f difsy3.f erel3.f extend.f qpmod3.f stabl3.f \
stablz.f start3.f subsys.f tperi.f trans3.f \
quad.f derqp4.f difsy4.f endreg.f erel4.f ichain.f newreg.f newsys.f \
qpmod4.f rchain.f rsort.f stabl4.f start4.f status.f trans4.f \
cfuncs.f chain.f chstab.f const.f cstab2.f cstab3.f cstab4.f cstab5.f \
derqp.f difsy1.f erel.f hpsort.f inclin.f invert.f ksphys.f physks.f \
qforce.f qpmod.f r2sort.f recoil.f redraw.f select.f slow.f stablc.f \
swcond.f switch.f transk.f transq.f transx.f vector.f xtf.f xtrnlu.f \
ycopy.f ysave.f \
absorb.f chaos2.f chdata.f chfind.f chfirr.f chinit.f chlist.f chmod.f \
chpert.f chpot.f chterm.f expel2.f fchain.f ghost.f giant2.f kcpert.f \
reduce.f reinit.f renew.f setsys.f tchain.f xcpred.f xtpert.f premsf.f \
circ.f spinup.f
```

Code Tests

Chaotic system   Exponential error growth

Small time-steps $\Delta t \propto R^{3/2}$

Two-body problem   Analytical solution

Evolution rate   Core collapse time

Mass segregation   Two mass groups

Systematic errors   Minimize energy drift

Performance   CPU time as $f(N)$